The Open University
Mathematics and Computing: A Fourth Level Course
M434 Differential Geometry

M434 AB

M434

Assignment Booklet

Contents **Cut-off date**

2 TMA M434 01 (Sections I.1–II.2 of *O'Neill*) 27 March 2001

5 TMA M434 02 (Sections II.3–III.5 of *O'Neill*) 29 May 2001

8 TMA M434 03 (Sections IV.1–V.3 of *O'Neill*) 24 July 2001

11 TMA M434 04 (Sections V.4–VI.2 and VI.6 of *O'Neill*) 18 September 2001

Instructions for TMAs

To save repetition, all *O'Neill*'s conventions will be used in TMA questions and may be used by you, without comment, when writing your solutions.

The usual way to write symbols that would be printed in bold is to underline them.

If a question says 'write down …', just the answer will be accepted. In all other cases, there are specific marks awarded for your working. We recommend showing your working for *all* questions. This will give your tutor the opportunity to award you some marks for a question where you have the working partially correct, even though you may not obtain the correct final answer.

For each TMA, please send your answers to all the questions to your tutor, along with an appropriately completed PT3 form. Note that all questions are used for assessment.

You will find instructions on how to fill in the PT3 form in the *Student Handbook*. Be sure to fill in the correct Assignment Number, e.g. for the first TMA this is

$\boxed{\text{M434 01}}$.

Do not forget that the cut-off date for a TMA represents the latest date by which your tutor should receive your answers, unless you have special permission, so allow for the time your answers will take to go through the post.

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3.1

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You should be able to answer Question 1 after you have studied Sections 1–5 of Part I.

Question 1 (25 marks)

The function $f : \mathbf{E}^3 \longrightarrow \mathbf{R}$, the point $\mathbf{p}$, the vector field V and the curve α are defined as follows:

$$f = xy + z^2,$$
$$\mathbf{p} = (-2, 1, 3),$$
$$V = x^2\, U_1 + xyU_2 - zU_3,$$
$$\alpha(t) = (-2\cos t + 4\sin t, \cos t - 2\sin t, 3(\cos t - \sin t)), \quad t \in \mathbf{R}.$$

The tangent vector $\mathbf{v_p}$ at $\mathbf{p}$ is defined by

$$\mathbf{v_p} = V(\mathbf{p}).$$

(i) Calculate the partial derivatives

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y} \quad \text{and} \quad \frac{\partial f}{\partial z},$$

and evaluate them at $\mathbf{p}$. [4]

(ii) Write down $\mathbf{v_p}$, and hence find, *using the definition*, the directional derivative $\mathbf{v_p}[f]$. [5]

(iii) Use the result of part (i) and Lemma I.3.2 of *O'Neill* to check your answer to part (ii). [3]

(iv) Express the directional derivative $V[f]$ in terms x, y and z. By making a suitable substitution, give an alternative calculation of $\mathbf{v_p}[f]$. Your answer should include a brief explanation of why you made the substitution that you did. [4]

(v) Find the composite

$$f(\alpha(t)).$$

Calculate the value of the derivative

$$(f(\alpha(t)))'$$

at $t = 0$. Compare your answer with that of part (ii), and explain the result of the comparison in terms of results proved in *O'Neill*. [5]

(vi) Find the differential df of f, expressing your answer in terms of the 1-forms dx, dy and dz. Hence find

$$df(V).$$

Explain, briefly, the relationship between your answers to this part and to part (iv). [4]

You should be able to answer Questions 2 and 3 after you have completed your study of Part I.

Question 2 (18 marks)

The function $F : \mathbf{E}^3 \longrightarrow \mathbf{E}^3$ and the function $G : \mathbf{E}^3 \longrightarrow \mathbf{E}^2$ are defined by

$$F = (x + y, y^2, yz),$$
$$G = (xz, yz).$$

(i) Find the Jacobian matrices representing the derivative maps F_* and G_*. *[3]*

(ii) Find the composite function $H = G(F)$ from $\mathbf{E}^3$ to $\mathbf{E}^2$. *[2]*

(iii) Evaluate the matrix representing G_* at F. *[2]*

(iv) Use the composite rule to find the Jacobian matrix representing the derivative map H_*. Confirm your result by direct calculation. *[4]*

(v) Find the set of points in $\mathbf{E}^3$ for which F is *not* regular. *[4]*

(vi) Find the image of the xz plane under F. *[3]*

Question 3 (12 marks)

The function $f : \mathbf{E}^3 \longrightarrow \mathbf{R}$, the 1-form ϕ and the 2-form η are defined by

$$f = xy - z^2,$$
$$\phi = y\,dx - x\,dy + z\,dz,$$
$$\eta = z\,dx\,dy + x^2\,dy\,dz + y\,dx\,dz.$$

Calculate the following.

(i) $df \wedge \phi$ *[3]*

(ii) $d\phi$ *[2]*

(iii) $d(f\phi)$ *[2]*

(iv) $d\eta$ *[3]*

(v) $df \wedge \eta$ *[2]*

You should be able to answer Question 4 after you have studied Section 1 of Part II.

Question 4 (17 marks)

The tangent vectors

$$\mathbf{e}_1 = (\ \ 1,\ \ 2,\ \ 2)/3,$$
$$\mathbf{e}_2 = (\ \ 2, -2,\ \ 1)/3,$$
$$\mathbf{e}_3 = (-2, -1,\ \ 2)/3$$

are all based at a point $\mathbf{p}$ of $\mathbf{E}^3$.

(i) Write down the attitude matrix, A, of this set of vectors. *[1]*

(ii) Show that $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ form a frame at $\mathbf{p}$. *[4]*

(iii) Write down the inverse A^{-1} of A. Explain briefly why your answer is correct. *[2]*

(iv) Evaluate $\mathbf{e}_1 \times \mathbf{e}_2 \cdot \mathbf{e}_3$, and hence determine whether the frame is right- or left-handed. *[3]*

(v) Express the tangent vector $\mathbf{v}_\mathbf{p} = (3, -6, 9)_\mathbf{p}$ at $\mathbf{p}$ as a linear combination of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$. *[4]*

(vi) Calculate $(\mathbf{e}_1 + 2\mathbf{e}_2 - 3\mathbf{e}_3) \cdot (3\mathbf{e}_1 - \mathbf{e}_2 + 7\mathbf{e}_3)$. *[3]*

You should be able to answer Question 5 after you have studied Section 2 of Part II.

Question 5 (16 marks)

The curve α is defined by

$$\alpha(t) = (\cos(e^t - 1), \sin(e^t - 1), e^t - 1), \quad t \in \mathbf{R}, \quad -1 < t < 1.$$

(i) (a) Find the velocity and speed of α at $\alpha(t)$. [3]

(b) Find the arc-length function $s(t)$ for α, based at $t = 0$. [3]

(c) Find a unit-speed reparametrization β of α, and confirm, by direct calculation, that β is unit-speed. [5]

(ii) Suppose that f is a one-one function from a set $D \subseteq \mathbf{R}$ to $\mathbf{R}$, that $0 \in \mathbf{R}$, that $f(0) = 0$ and that γ is a curve defined by

$$\gamma(t) = (\cos(f(t)), \sin(f(t)), f(t)), \quad t \in D.$$

Find a unit-speed reparametrization of γ. [5]

Question 6 is concerned with differential forms. It makes use of the explanation of the action of a wedge product of 1-forms on a pair of tangent vectors as discussed in the Commentary for Section I.6.

Question 6 (12 marks)

(i) The functions ϕ and ψ from the set of all tangent vectors at all points in $\mathbf{E}^3$ to the real numbers are defined, with the usual notation, by

$$\phi(\mathbf{v_p}) = 2\mathbf{v} \cdot 3\mathbf{p},$$
$$\psi(\mathbf{v_p}) = v \cdot v.$$

(a) Exactly one of ϕ and ψ is a 1-form. Decide which is, and which is not, a 1-form, justifying your answers. [4]

(b) For the function which is a 1-form, evaluate it on U_1, U_2 and U_3, and hence express it in terms of the 1-forms dx, dy and dz. [3]

(ii) The 1-form ζ on $\mathbf{E}^3$ is defined by

$$\zeta = z\,dx + x\,dy + y\,dz.$$

Show that if $\mathbf{v_p}$ and $\mathbf{w_p}$ are tangent vectors at $\mathbf{p}$, then

$$d\zeta(\mathbf{v_p}, \mathbf{w_p}) = (dx + dy + dz)((\mathbf{v} \times \mathbf{w})_{\mathbf{p}}).$$ [5]

You should be able to answer Question 1 after you have studied Section 3 of Part II.

Question 1 (20 marks)

The curve α, with Frenet apparatus T, N, B, κ and τ, is defined by

$$\alpha(t) = \tfrac{1}{3}(2\cos t - \sin t, \cos t + 2\sin t, 2t), \quad t \in \mathbf{R}.$$

(i) Show that α is unit-speed. *[4]*

(ii) Find the Frenet apparatus of α. *[12]*

(iii) Explain what the results obtained in part (ii) tell you about the curve α. *[4]*

You should be able to answer Question 2 after you have studied Sections 3–4 of Part II.

Question 2 (20 marks)

The curve α, with Frenet apparatus v, T, N, B, κ and τ, is defined by

$$\alpha(t) = (2\cosh t, \sinh t, 2t), \quad t \in \mathbf{R}.$$

(i) Find the Frenet apparatus of α. *[15]*

(ii) Find the Frenet frame for α at $\alpha(0)$. *[3]*

(iii) Find the vector

$$\mathbf{u} = (T + 2B)/\sqrt{5},$$

and hence show, directly from the definition, that α is a cylindrical helix. State, with a brief reason, whether or not α is a circular helix. *[2]*

You should be able to answer the remaining questions when you have studied as far as Section 5 of Part III.

Question 3 (20 marks)

(i) This part concerns the curve β, with Frenet apparatus v_β, T_β, N_β, B_β, κ_β and τ_β, and also the curve α that you investigated in Question 2 above. You will need the results from Question 2.

The speed and curvature of β are the same as those of α, the torsion of β is minus the torsion of α, $\beta(0) = (2, 2, -1)$ and the Frenet frame of β at $t = 0$ is

$$T_\beta(0) = (1, 0, -1)/\sqrt{2},$$
$$N_\beta(0) = (1, 0, 1)/\sqrt{2},$$
$$B_\beta(0) = (0, 1, 0).$$

(a) Explain why α and β are congruent. *[1]*

(b) Construct an isometry F such that $F(\alpha) = \beta$. You should give F in the form TC, specifying both the orthogonal part C and the translation part T. *[7]*

(c) Write down $\beta(t)$ in terms of t. *[2]*

(ii) For each of the following mappings $F : \mathbf{E}^3 \longrightarrow \mathbf{E}^3$, state whether or not it is an isometry of $\mathbf{E}^3$, and give a reason for your answer.

(a) $F = (x, x, y)$ *[2]*

(b) $F = (x^2, y^2 + 1, z^2)$ *[2]*

(c) $F = ((x - z)/\sqrt{2}, (x + y + z)/\sqrt{3}, (x - 2y + z)/\sqrt{6})$ *[2]*

(d) $F = (x + 1, y - x, 4 - z)$ *[2]*

(e) $F = (2 - x, 3 - y, 4 - z)$ *[2]*

Question 4 (10 marks)

A family of curves α_k, $k \in \mathbf{R}$, is defined as follows:

$$\alpha_k(t) = (\cos t, \sin t, kt), \quad t \in \mathbf{R}.$$

The curvature and torsion functions of α_k are

$$\kappa_k = \cancel{\frac{\sqrt{1+k^2}}{k^3}}, \qquad \kappa_k = \frac{1}{1+k^2}$$

$$\tau_k = \frac{k}{1+k^2}.$$

(i) Establish the following results.

(a) All curves in the family are circular helices. *[2]*

(b) Exactly one curve in the family is a plane curve. *[2]*

(c) If α_k is *not* planar, then α_k is congruent to exactly one other curve in the family. *[3]*

(ii) If α_k is not planar, find the isometry whose existence is guaranteed by the result in part (i)(c) above. (You should not need to calculate any Frenet frames for this part.) *[3]*

Question 5 (10 marks)

The vector fields V and W and the function f are defined by

$$\begin{aligned} V &= xyU_1 + zU_2 - y^2U_3, \\ W &= z^2U_1 - (xyz)U_3, \\ f &= xy - z^2. \end{aligned}$$

(i) Calculate $V[f]$. [3]

(ii) Calculate $\nabla_V W$. [4]

(iii) Calculate $\nabla_V fW$. [3]

Question 6 (10 marks)

The vector fields F_1, F_2 and F_3 are defined by

$$\begin{aligned} F_1 &= (\cos f)\, U_1 - (\sin f)\, U_3, \\ F_2 &= (\sin f)\, U_1 + (\cos f)\, U_3, \\ F_3 &= -U_2, \end{aligned}$$

where f is a differentiable function from $\mathbf{E}^3$ to $\mathbf{R}$.

(i) Show that F_1, F_2 and F_3 form a frame field. [3]

(ii) Find the dual 1-forms for F_1, F_2 and F_3, giving your answer in terms of the differentials dx, dy and dz. [2]

(iii) Find the matrix ω of connection forms for F_1, F_2 and F_3, giving your answer in terms of the differential df of f. [5]

Question 7 (10 marks)

The curve α is a unit-speed circular helix, with Frenet apparatus T, N, B, $\kappa > 0$ and τ.

The curve β is defined by

$$\beta(t) = \alpha(t) + T(t),$$

with Frenet apparatus v_β, T_β, N_β, B_β, κ_β and τ_β.

(i) Find κ_β and τ_β in terms of κ and τ. [8]

(ii) Show that β is a plane curve if and only if α is a plane curve. [2]

You should be able to answer Question 1 after you have studied Sections 1–3 of Part IV.

Question 1 (25 marks)

This question concerns the function $f : \mathbf{E}^3 \longrightarrow \mathbf{R}$ defined by

$$f = -x^2 + y^2 + 2z$$

and the mapping $\mathbf{x} : \mathbf{E}^2 \longrightarrow \mathbf{E}^3$ defined by

$$\mathbf{x}(u, v) = (u + v, u - v, 2uv).$$

The set of points M is defined by

$$M : f = 0.$$

(i) Show that M is a surface in $\mathbf{E}^3$. [3]

(ii) Show that $\mathbf{x}$ is a coordinate patch. [5]

(iii) Show that $\mathbf{x}$ covers the whole of M, by carrying out the following steps.

(a) Show that the image $\mathbf{x}(u, v)$ lies in M for all values of u and v.

(b) Show that each point $\mathbf{p}$ of M can be expressed in the form

$$\mathbf{p} = \mathbf{x}(u_0, v_0)$$

for suitable values of u_0 and v_0. [5]

(iv) (a) Find values of u_0 and v_0 such that

$$(1, 5, -12) = \mathbf{x}(u_0, v_0).$$

(b) Show that the tangent vector

$$\mathbf{v_p} = (5, 1, 0)_{(1,5,-12)}$$

is tangent to the surface M. [4]

(v) Let g be the function defined on M by

$$g = x + 2y - z^2.$$

(a) Express g as a function of u and v.

(b) Show that

$$\mathbf{x}_u(u, v)[g] = 3 - 8uv^2,$$

and find a similar expression for

$$\mathbf{x}_v(u, v)[g].$$ [5]

(vi) Calculate

$$\mathbf{v_p}[g],$$

where $\mathbf{v_p}$ is as defined in part (iv). [3]

You should be able to answer Question 2 after you have studied Section 5 of Part IV.

Question 2 (25 marks)

This question concerns the surface of revolution M parametrized by

$$\mathbf{x}(u,v) = ((3+\sin u)\cos v, (3+\sin u)\sin v, \cos u),$$

the sphere S parametrized by

$$\mathbf{y}(u,v) = (\sin u, \cos u\cos v, \cos u\sin v)$$

and the mapping F from M to S defined by

$$F : \mathbf{x}(u,v) \longmapsto \mathbf{y}\left(\frac{\pi}{2} - u, \frac{\pi}{2} + v\right).$$

(i) Find the partial velocities on both M and S. [4]

(ii) Construct unit normal vector fields U on M and V on S. Your answers should be in the form

$$U(\mathbf{x}(u,v)) = \ldots$$

etc. [4]

(iii) Find $F_*(\mathbf{x}_u(u,v))$ and $F_*(\mathbf{x}_v(u,v))$, expressing your answers in terms of $\mathbf{y}_u$ and $\mathbf{y}_v$ at a suitable point of S. [5]

The 2-form A on M is defined by

$$A(\mathbf{v}_1,\mathbf{v}_2) = \mathbf{v}_1 \times \mathbf{v}_2 \cdot U(\mathbf{p}),$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are tangents to M at $\mathbf{p}$.

The 2-form B on S is defined by

$$B(\mathbf{w}_1,\mathbf{w}_2) = \mathbf{w}_1 \times \mathbf{w}_2 \cdot V(\mathbf{p}),$$

where $\mathbf{w}_1$ and $\mathbf{w}_2$ are tangents to S at $\mathbf{p}$.

(iv) Evaluate

(a) $A\left(\mathbf{x}_u(u,v), \mathbf{x}_v(u,v)\right)$,

(b) $B\left(\mathbf{y}_u(u,v), \mathbf{y}_v(u,v)\right)$. [6]

(v) Calculate

$$\left(F^*B\right)\left(\mathbf{x}_u(u,v), \mathbf{x}_v(u,v)\right),$$

and hence show that

$$F^*B = g(u,v)A,$$

where $g(u)$ is a function of u only. Write down g. [6]

You should be able to answer Questions 3 and 4 after you have Section 3 of Part V.

Question 3 (25 marks)

The surface M is parametrized by the mapping

$$\mathbf{x}(u,v) = (\sqrt{1+u^2}\cos v,\ \sqrt{1+u^2}\sin v,\ \sinh^{-1} u), \qquad u, v \in \mathbf{R}.$$

(i) Find the partial velocities, and hence construct a unit normal vector field U on M. [4]

Let S be the shape operator on M derived from U.

(ii) Working from Definition V.1.1 of *O'Neill*, find

$$S(\mathbf{x}_u(u,v)) \quad \text{and} \quad S(\mathbf{x}_v(u,v)),$$

giving your answers in terms of $\mathbf{x}_u(u,v)$ and $\mathbf{x}_v(u,v)$. Hence find the normal curvatures in the directions of the partial velocities. [7]

(iii) Write down the matrix representing S with respect to the basis of partial velocities $\{\mathbf{x}_u(u,v), \mathbf{x}_v(u,v)\}$ at $\mathbf{x}(u,v)$. Hence find the Gaussian and mean curvature functions on M. [3]

(iv) Write down the principal curvatures of M at $\mathbf{x}(u,v)$, giving a brief reason for your answers. Does M have any umbilic points? Justify your answer briefly. [4]

(v) Write down the eigenvectors of S at $\mathbf{x}(u,v)$, and verify directly that they are orthogonal. [4]

(vi) How many asymptotic directions are there at each point of M? Justify your answer briefly. [3]

Question 4 (25 marks)

Let β be a unit-speed curve with Frenet apparatus T, N, B, $\kappa > 0$ and τ, where the torsion is *constant*. The surface M is parametrized by the mapping

$$\mathbf{x}(u,v) = \beta(u) + vB(u),$$

where u and v are suitably restricted so that $\mathbf{x}$ does parametrize M.

(i) Find the partial velocities for this parametrization, and hence express a normal vector field U on M in terms of the parameters and the Frenet apparatus of β. [5]

Let S be the shape operator on M derived from U.

(ii) Calculate

$$S(\mathbf{x}_u(u,v)) \quad \text{and} \quad S(\mathbf{x}_v(u,v)),$$

giving your answers in terms of $\mathbf{x}_u(u,v)$ and $\mathbf{x}_v(u,v)$. Hence find the matrix representing S with respect to the partial velocities. [6]

(iii) Write down the Gaussian curvature, K, and the mean curvature, H, for M. [3]

(iv) For each of the attributes 'flat' and 'minimal', is it possible for M to have that attribute? If not, explain why not; if so, explain any necessary restrictions on the nature of the curve β. [5]

(v) (a) Find the principal curvatures for M at $\mathbf{x}(u,v)$. You need not simplify your expressions. [2]

(b) Is it possible for M to have umbilic points? If so, find them all; if not, explain why not. [4]

TMA M434 04 **Cut-off date** 18 September 2001

Your attention is drawn to Section 9.6 of the *Student Handbook*, which reads as follows: 'So that scores can be recorded and documentation prepared at the end of each course, *no extension to the cut-off date will normally be allowed for a course's last assignment.*'

You should be able to answer Question 1 after you have studied Sections 4–5 of Part V.

Question 1 (25 marks)

The surface M is parametrized by the mapping

$$\mathbf{x}(u,v) = (v\sinh u, v\cosh u, v), \quad v > 0, \quad u, v \in \mathbf{R}.$$

(i) Find the partial velocities, and hence construct a unit normal vector field U on M. [4]

(ii) Using the field U constructed in part (i), find the functions E, F, G, l, m and n on M. [7]

(iii) Find the Gaussian, mean and principal curvature functions on M. [6]

(iv) How many asymptotic directions does M have at each point? Justify your answer. [3]

(v) Are the u and v parameter curves geodesics in M? Justify your answer. [5]

You should be able to answer Question 2 after you have studied to the end of Part V.

Question 2 (20 marks)

A surface M is parametrized by the Monge patch

$$\mathbf{x}(u,v) = (u, v, f(u,v))$$

for a function $f : \mathbf{R}^2 \longrightarrow \mathbf{R}$. The curve α in M has coordinate functions α_1 and α_2, that is,

$$\alpha(t) = \mathbf{x}(\alpha_1(t), \alpha_2(t)).$$

(i) Express a unit normal vector field U on M in terms of the partial derivatives f_u and f_v of f. [5]

(ii) Express the velocity α' of α in terms of the derivatives of α_1, α_2 and f, and hence show that

$$\alpha'' = (\alpha_1'', \alpha_2'', f_{uu}\alpha_1'^2 + 2f_{uv}\alpha_1'\alpha_2' + \overset{f_{vv}}{\cancel{f_v}}\alpha_2'^2 + f_u\alpha_1'' + f_v\alpha_2''),$$

where the derivatives of f are evaluated at $(\alpha_1(t), \alpha_2(t))$, and α_1 etc. are evaluated at t. [6]

(iii) (a) Determine whether or not the curve $\alpha(t) = \mathbf{x}(t,t)$ is a geodesic of the surface parametrized by

$$\mathbf{x}(u,v) = (u, v, u^2 + v^2).$$ [4]

(b) Determine whether or not the curve $\alpha(t) = \mathbf{x}(t^2, t^2)$ is an asymptotic curve of the surface parametrized by

$$\mathbf{x}(u,v) = \left(u, v, \tfrac{1}{2}(u^2 - v^2)\right).$$ [5]

You should be able to answer Questions 3–5 after you have completed your study of Part VI.

Question 3 (25 marks)

Let M be a surface parametrized by a single patch $\mathbf{x}$ for which $E = 1$ and $F = 0$.

(i) Define a tangent frame field on M in terms of the partial velocities. [4]

(ii) Find the dual 1-forms θ_1 and θ_2 for E_1 and E_2, expressing your answers in terms of du and dv. [4]

(iii) By using the first structural equations, show that

$$\omega_{12} = \frac{\partial}{\partial u}\left(\sqrt{G}\right) dv.$$

[6]

(iv) Use the second structural equation to show that

$$\frac{\partial^2}{\partial u^2}\left(\sqrt{G}\right) + K\sqrt{G} = 0.$$

[5]

(v) *Hence* find the Gaussian curvature function for the surface parametrized by

$$\mathbf{x}(u, v) = \left(\frac{u}{(1+v^2)^{1/2}}, v, \frac{uv}{(1+v^2)^{1/2}}\right).$$

[6]

Question 4 (20 marks)

A unit-speed curve α has Frenet apparatus T, N, B, $\kappa > 0$ and τ. The surface M is parametrized by the mapping $\mathbf{x}$ defined by

$$\mathbf{x}(u, v) = \alpha(u) + vT(u), \quad v > 0,$$

where u and v are further restricted sufficiently to ensure that $\mathbf{x}$ *is* the parametrization of a surface.

(i) Find expressions for the functions E, F and G. [5]

(ii) Express a unit normal vector field on M in terms of the Frenet apparatus of α. [4]

(iii) Find the Gaussian, mean and principal curvature functions for M. What can you deduce about the surface M? [4]

Let α be the unit-speed reparametrization of the curve defined by

$$\beta(t) = (\cosh t, \sinh t, t), \quad t \in \mathbf{R}.$$

(iv) Show that the corresponding surface cannot have planar points. [7]

Question 5 (10 marks)

The surface M is covered by a single orthogonal patch for which

$$E = \cosh^2 u, \quad G = 1, \quad F = 0.$$

By considering a suitable tangent frame field E_1, E_2, and the corresponding dual 1-forms θ_1, θ_2, show that M is flat. [10]

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